

Keynote for Conference on “Why and How We Give and Ask for Reasons”
Hradec Králové—October 19-22, 2021

Reasoning, Reason Relations, and Semantic Content¹

I. Normative Pragmatics: From Reasoning Practices to Relations of Implication and Incompatibility

A familiar semantic order of explanation, inspired by Frege, begins with the distinction between two truth values, true and false. It then seeks to explain what is truth evaluable, what can *have* those truth values—what is expressed by what we can for that reason understand syntactically as *declarative sentences*—in terms of truth (or falsity) *conditions*. These are thought of as states of the world that in a distinctive semantic sense *make* the truth-evaluable sentences true or false. For that reason they are intelligible as what the sentences count as representing. In a suitable semantic metavocabulary, the truth conditions of, or truth-evaluable contents expressed by, declarative sentences take the mathematical form of functions from represented worldly states to the truth values of sentential representings of them.

¹ In this paper I deploy a number of arguments and report a number of results due to Ryan Simonelli (simonelli@uchicago.edu), Ulf Hlobil (ulf.hlobil@concordia.ca), and Dan Kaplan (dan.kaplan@pitt.edu), who are members of our logic working group “Research on Logical Expressivism.” I mark their contributions as best I can along the way, to indicate what they are responsible for. They should not be presumed to endorse the use I have made of their work here.

The core task of *pragmatics* is offering an account of what one is doing in saying or thinking something expressible by the *use* of declarative sentences. In this semantics-first order of explanation, pragmatics is thought of as explanatorily downstream from the representational semantic story. The individual abilities exercised, or the social practices engaged in when speakers and thinkers use sentences with truth-*evaluable* contents or meanings are to be understood in terms of truth-*evaluating* practical attitudes. *Taking-true*, in practice treating a sentence as expressing something true, is doxastically *accepting* it. *Taking-false*, in practice treating a sentence as expressing something false, is doxastically *rejecting* it. Those doxastic stances or practical attitudes can be manifested publicly by using sentences to perform speech acts of asserting and denying. It follows that what can be asserted or denied, doxastically accepted or rejected, is just what can take truth values.

A converse, *pragmatics-first* order of explanation begins with an account of the practical attitudes of doxastic acceptance and rejection, and seeks to understand in terms of them what is said or expressed by the declarative sentences used in the speech acts of assertion and denial that manifest those attitudes publicly.

I think the best strategy for developing such a pragmatist semantics, a use-theory of meaning, do not lie in simply standing the traditional semantics-first story on its head. One can agree that doxastic acceptance can be characterized as taking-true and doxastic rejection as taking-false without treating those characterizations in terms of truth-evaluation as of use in substantial semantic explanations of what it is that interlocutors accept or reject: what is expressed by the declarative sentences they use to say something in the sense of asserting or denying it.

Instead of understanding semantic content representationally, in terms of truth, one can look to further essential features of the discursive practices within which performances can have the pragmatic significance of assertions and denials, expressing doxastic practical attitudes of acceptance and rejection. For any autonomous discursive practice—any language game one can play though one plays no other—must include not only the making of claims, manifesting doxastic acceptance or rejection of them, but also practices of *challenging* and *defending* those claims by giving reasons for and against them. So another strategy would be to try to understand what is said or claimed, the contents that can be accepted or rejected doxastically, in terms of those reason relations among claimables: relations of being a reason for or against.

The claim is not that there cannot be acceptance and rejection in the absence of practices of giving reasons for and against the adoption of those attitudes. One might simply tick off one box rather than another, as on a ballot, menu, or scorecard. But such indications of preference for one or another option presuppose specifications of the contents of those options. The issue is what is required for that. The claim is that if one wants to understand *what* can be accepted or rejected, one should look to a fuller discursive context that includes practices of defending and challenging those attitudes by giving reasons for and against them.

The idea is to proceed in two stages in understanding *semantics* in terms of *pragmatics*, *meaning* in terms of *use*. The first stage begins with practices of reasoning: practices of defending and challenging commitments to accept and reject, undertaken in the first instance by performing speech acts of asserting and denying. On the basis of an account of such reasoning practices involving *claimings*, it offers an account of the *reason relations* among *claimables* (what can be doxastically accepted or rejected) in virtue of which some *claimings* can serve as reasons for and against others. The second stage then elaborates a semantic understanding of the contents that can be doxastically accepted or rejected in terms of roles *claimables* expressed by declarative sentences play in those reason relations. The core of such a two-phase, pragmatics-first semantic explanatory strategy is to use reason relations among claimable contents to mediate between a *pragmatic* account of what discursive practitioners *do* in making claims and giving and asking for reasons for them, on the one hand, and a *semantic* account of the claimable contents they assert and deny, and defend and challenge by engaging in such practices, on the other hand.

Here is a sketch of how the first stage of such an account of the path from pragmatics to semantics might go. We can unpack the distinction and relation between practices of reasoning about *claimings* (doxastic acceptances and rejections), and reason relations among *claimable* contents into these pieces:

1. Discursive practice as such involves reasoning. For in addition to accepting and rejecting what is expressed by declarative sentences, interlocutors must be able both to *defend* and to *challenge* the rational credentials of those stances or practical attitudes.
2. Defending (the credentials of) a claiming is producing further claimings that provide reasons *for* the acceptance or rejection being challenged. Challenging (the credentials of)

a claiming is producing further claimings that provide reasons *against* the acceptance or rejection being challenged.

3. If accepting A functions practically as a reason *to accept* B, then A provides a reason *for* B, and if accepting A functions practically as a reason *to reject* B then A provides a reason *against* B. Reason relations are relations that one set of claimables stands in to another when the first consists of reasons for or against the other.
4. We can call these reason relations “implication” and “incompatibility.” To give a reason *for* is to commit oneself to accept premises that *imply* the claimable a reason is being given for. To give a reason *against* is to commit oneself to accept premises that are *incompatible* with the claimable a reason is being given against.²

To say symbolically that a set Γ of acceptables/rejectables *implies* acceptable/rejectable A, we can write “ $\Gamma \vdash A$.” Use of the “snake turnstile” rather than the more familiar double turnstile \models of semantic consequence or the single turnstile \vdash of derivability reminds us that we are concerned with *rational* implications, not specifically *logical* implications. This is the sense in which “Pedro is a donkey” implies “Pedro is a mammal.” Because the goodness of that implication depends on the contents of the nonlogical concepts donkey and mammal, rather than solely on the contents of logical concepts such as those expressed by conditionals and negation, Wilfrid Sellars calls these “materially” good implications.

We can understand an implication as *logically* good in case it meets two conditions: i) it is materially good, and ii) it’s material goodness is robust under arbitrary uniform substitution of nonlogical vocabulary for nonlogical vocabulary. We logical expressivists understand logical vocabulary as demarcated by a distinctive expressive role, whose paradigm is the way conditionals let us make explicit implications and negation lets us make explicit incompatibilities. But the story I am telling here addresses considerations that arise upstream of the introduction of specifically logical vocabulary to codify material reason relations.³ For the other basic kind of reason relation, to say that a set Γ of acceptables/rejectables is *incompatible* with acceptable/rejectable A, we can write “ $\Gamma \# A$.”

² These are the base cases that pragmatically define implication and incompatibility. On this basis more sophisticated practices can be built. One example would be giving a reason against a claim by rejecting some claimable that implies it.

³ For a sketch of how this subsequent story goes, see my “From Logical Expressivism to Expressivist Logics” *Nous: Philosophical Issues*, Volume 28, Issue 1, October 2018, (a volume devoted to the philosophy of logic), pp. 70-88, reprinted in Ondrej Beran, Vojtech Kolman, Ladislav Koren (eds.) *From Rules to Meaning: New Essays on Inferentialism* (Routledge Studies in Contemporary Philosophy).

A promising direction in which such an account might be deepened and extended is suggested by Greg Restall and David Ripley's *bilateralist* normative pragmatic analysis of implication relations. They suggest that we understand the implication statement " $\Gamma \vdash \sim A$ " as saying that that the position in which one is committed to *accept* all of Γ and *reject* A is normatively out of bounds. This philosophically powerful pragmatic interpretation allows them to understand sequent calculi as consisting of rules that tell us that if some positions are out of bounds, then some others are also.

(From their point of view, a principal benefit of the account is that it makes sense of multiple conclusion implications of the sort Gentzen introduces for classical logic. For we can say that Γ implies Δ just in case commitment to accept everything in Γ and reject everything in Δ is out of bounds. For the moment I ignore multisuccedent sequents, though they will become relevant to my story when I discuss the transition to semantics Part Two.)

This bilateralist reading of implication understands the role of reason relations to be articulating norms that govern the adoption of practical doxastic attitudes of acceptance and rejection. They guide and constrain what interlocutors do by dividing constellations of attitudes into those that are appropriate and inappropriate (their "in bounds" and "out of bounds"), rather than by issuing imperatives that determine at any point what one must do.

The normative pragmatic role of reason relations of implication and incompatibility can be further elaborated by thinking about reasoning practices in terms of commitments and entitlements. Here the basic claim is that to be intelligible as practices of *reasoning*, in the sense of accepting and rejecting claimables and defending and challenging those stances with reasons for and against them, the participants in such practices must be understood as keeping track of two different sorts of normative status: the kind of *commitment* one undertakes or acknowledges in accepting or rejecting a claimable by asserting or denying a sentence expressing it, and the sort of *entitlement* to that status or practical attitude that is at issue when *reasons* are offered for or against it. Accepting or rejecting a claimable, paradigmatically by asserting or denying it, is taking a stand on it, adopting a stance towards it. It is committing oneself with respect to it, in the way one would by saying "Yea" or "Nay" to it in response to a suitable yes/no question.

What difference does it then make whether an interlocutor can offer reasons to accept what he has accepted or to reject what he has rejected? The *commitments* involved, the stances taken up, the attitudes adopted, are the same either way. But it is also an integral feature of specifically *doxastic* commitments that one's *entitlement* to those commitments is always potentially at issue. For in taking up a doxastic stance one renders oneself liable to demands for justification, for exhibition of reasons to accept or reject the claim one has accepted or rejected. One may be challenged to show that the position one has adopted is normatively appropriate, "in bounds," a commitment one was entitled to adopt.

Reasons matter because other practitioners must distinguish between the acceptances and rejections the speaker in question is *entitled* to, in virtue of having *reasons* to adopt those attitudes, and those the speaker is *not* entitled to, because unable to defend those commitments by offering reasons when suitably challenged to do so. It follows that for each interlocutor there must be not only a difference between the attitudes (commitments) he has adopted and those he has not, but also, within those he has adopted, between those he is entitled to or justified in, has rational credentials for, and those that are *mere* commitments, bare of such accompanying entitlements. In *Making It Explicit* I argue that part of what turns practically on one's entitlement or justification—the second-person correlate of the first-person *responsibility* to defend one's commitments when one's entitlement is suitably challenged—is the testimonial *authority* of one's act: its capacity to license others to adopt a corresponding attitude. The essential point is that in addition to the *committive* dimension of assertional practice, there is the *critical* dimension: the aspect of the practice in which the rational propriety of those commitments, their justificatory status, is assessed. I implicitly invoked this dimension of entitlement earlier when I talked about defending and challenging *the credentials* of commitments.

(The claim that the autonomous discursive practices in which some performances can have the significance of the undertaking of specifically *doxastic* commitments must include the in-principle liability of such commitments to challenges to their associated entitlements is entirely compatible with understanding such practices as built around a default-and-challenge structure, in which commitments count as in order until and unless suitably challenged by undertaking commitments that offer reasons against them.)

Restall-Ripley bilateralism explains implication in terms of a single pragmatic normative status: a constellation of acceptances and rejections being “in bounds,” or, contrastingly, “out of bounds,” appropriate or inappropriate, OK or not OK. Distinguishing the two normative statuses of commitment and entitlement and their contrasting statuses permits us to discern further fine structure. In these terms, to say that a constellation of acceptances and rejections is “out of bounds” is to say that it is a collection of *commitments* to which one cannot be jointly *entitled*. In terms of commitments and entitlements, we can lay alongside their analysis of implication the analysis of incompatibility I offer in *Making It Explicit*: two commitments are incompatible when commitment to one precludes entitlement to the other. Ryan Simonelli has nicely synthesized this understanding of incompatibility with the Restall-Ripley understanding of implication in the definitions:

5. Γ implies A ($\Gamma|\sim A$) just in case commitment to accept everything in the premise-set Γ precludes entitlement to *reject* A.
6. Γ is incompatible with (rules out) A ($\Gamma\#A$) just in case commitment to accept everything in premise-set Γ precludes entitlement to *accept* A.

Like the original normative pragmatic bilateral account of implication, these principles make explicit what it is that practitioners need to *do* in order thereby practically to be taking or treating some claimables to imply or be incompatible with others. They need to take or treat some *commitments* as precluding *entitlement* to others, in keeping deontic score on their own and others’ normative statuses.

This more articulated bilateral account of the normative pragmatic functional roles that relations among the acceptables/rejectables expressed by declarative sentences must play in order for them to count as *reason* relations of implication and incompatibility can be connected to the prior discussion of how such reason relations are intelligible as providing reasons *for* and *against* commitments in practices of defending and challenging them by the following principles:

7. Any set of commitments that *precludes entitlement to reject* A thereby *implicitly* commits one to *accept* A.
8. Any set of commitments that *precludes entitlement to accept* A thereby *implicitly* commits one to *reject* A.

We can think of these principles as codifying definitions of a concept of some commitments being *implicit* in others. In the case of implication, they are “implicit in” in the literal sense of “implied by” a premise-set. Here that fundamental, etymologically natural notion of implicitness is being extended to include reason relations of incompatibility, on the basis that the pragmatic definitions (5) and (6) of implication and incompatibility show them to be two species of one genus. On this account, a reason *against* a *rejection* is an implication of acceptance of that conclusion, since “ $\Gamma|\sim A$ ” says that commitment to all of Γ precludes entitlement to reject A . That is the same as a reason *for* an *acceptance*. Dually, an incompatibility $\Gamma\#A$ exhibits its premises Γ as providing both a reason *against* acceptance and (so) a reason *for* rejection.

Principles (1) through (8) outline an order of explanation that begins with a characterization of practices of making claims and defending and challenging them, and ends with a specification of the functional role relations among the contents that are accepted or rejected, defended and challenged must play in order properly to be understood as relations of implication and incompatibility among those claimables. This is the first step in the two-stage pragmatics-first strategy for understanding semantic content. The second step is then to show how to understand the *semantic contents* of the declarative sentences used to assert and deny in terms of reason relations of implication and incompatibility among those claimables. We turn next to that task.

II. Truth-Maker Semantics and Reason Relations

The idea is that once we have an understanding from the side of pragmatics of the fundamental pair of opposite-but-complementary reason relations, implication and incompatibility, it will be possible to use them to formulate a *semantic theory* explicating the acceptables/rejectables expressed by declarative sentences. Understanding what can be doxastically accepted or rejected in terms of the roles declarative sentences can play in reason relations of implication and incompatibility would provide a purely *pragmatic* explication of a fundamental *semantic* concept: the concept of the *contents* expressed by those declarative sentences. What I want to do next is to explain two contributions to this enterprise that are made by recent work by two other members of the ROLE working group, Ulf Hlobil and Dan Kaplan. Hlobil offers an illuminating perspective on the relation between a pragmatic story along the lines I have been telling here and the best contemporary work in formal semantics. Kaplan shows in detail how a proper semantic account of the contents expressed by declarative sentences can be elaborated from the role those sentences play in reason relations of implication and incompatibility.

One of the most sophisticated, flexible, and expressively powerful formal semantic understandings of conceptual content available today is Kit Fine's truth-maker semantics.⁴ It is built on a space of what he calls "states." We are invited to think of the states as facts or situations, but the notion is an adaptable one, sufficiently general to include whatever it is that we might think of as making declarative sentences true or false. A subset of the space of states is distinguished as the *possible* states. The only structure imposed on the state space is a partial ordering of part-hood: some states are parts of others. It is assumed that every subset of the space has a least upper bound. It can be thought of as the *fusion* of the elements of the subset: the unique whole of which they are all parts. The content or proposition expressed by a sentence

⁴ Introduced in "A Theory of Truth-maker Content I: Conjunction, Disjunction, and Negation" *Journal of Philosophical Logic* (2017) 46:625-674.

A is then specified bilaterally, as a pair of sets of states: those “verifying” states that would make it true and those “falsifying” states that would make it false.

Like intensional semantics appealing to possible worlds, truth-maker semantics advances from the fundamental opposition of truth and falsity to a notion of content as truth conditions. It is more general in including also a notion of falsity conditions, which are not assumed in general to be uniformly computable from the truth conditions. Its basic notion of a state is more capacious than that of possible world.

Possible worlds are included as special cases of states. For two states can be defined as *compatible* just in case their fusion is one of the states distinguished as *possible*. And a state can be understood as a possible world just in case it is a maximal possible state, in the sense of containing as parts every state compatible with it.

Further flexibility (in the form of hyperintensionality) is secured by not restricting the state space to *possible* states, but embedding those in a larger structure that includes multiple distinct *impossible* ones. In addition, the mereological structure of the state space provides expressive resources in the truth-maker semantic metavocabulary that have no analogue in classical possible worlds semantics. The bilateral conception of content, including falsifiers as well as verifiers and not assuming that either sort of semantic interpretant can straightforwardly be computed from the other, turns out to pay large expressive dividends.

The truth-maker semantic framework permits various definitions of the reason relations of implication and incompatibility. As state t counts as incompatible with a set S of states just in case the fusion of it with all the states in S is an impossible state. We can then say that $\Gamma \# A$ just in case any fusion of verifiers of all the members of Γ with any verifier of A is an impossible state. On the side of implication, there are a number of different notions of semantic consequence definable in the truth-maker setting, and Fine considers it a signal virtue of his approach that it can express and compare such a variety of senses of “follows from.” For instance, Γ verifier-entails A in case every state that verifies all the sentences of Γ verifies A .

Ulf Hlobil shows how the truth-maker framework allows the definition of a further notion of implication, which Fine does not consider.⁵ We can say that

9. $\Gamma \mid \sim \Delta$ iff any fusion of a state that verifies all the members of Γ with a state that falsifies all the members of Δ is an impossible state.

He invites us to compare this semantic notion of multisuccedent implication with Restall and Ripley’s bilateral pragmatic notion. Recall that they understand

10. $\Gamma \mid \sim \Delta$ iff any position that includes accepting all of Γ and rejecting all of Δ is normatively incoherent or “out of bounds”—as we have read it: one cannot be entitled to such a constellation of commitments.

Both conceptions can be thought of as stemming from the same intuition that led C. I. Lewis to define his notion of strict implication by saying that in this sense of “implies” A implies B in case it is *impossible* for A to be true and B to be false.

(It is the strengthening by necessitation of the horseshoe of bivalent classical logic.)

It is clear that these are isomorphic understandings of implication. The role played in the truth-maker semantic definition by verifiers and falsifiers of sentences is played in the bilateral pragmatic definition by practical attitudes of acceptance and rejection of sentences. And the role played in the truth-maker semantic definition by the impossibility of the state that results from fusing those verifiers and falsifiers is played in the bilateral pragmatic definition by the normative incoherence (or “out of bounds-ness”) of the position that results from concomitant commitment to those acceptances and rejections.

The isomorphism extends to incompatibility as well as implication. In the single-succedent formulation, we can lay alongside the truth-maker semantic reading:

11. $\Gamma \# A \Leftrightarrow$ the state resulting from *fusion* of any *verifiers* of all the members of Γ with any *verifier* of A is an *impossible* state,

the normative pragmatic reading:

12. $\Gamma \# A \Leftrightarrow$ the position resulting from *concomitant commitment* to *accept* all of Γ and to *accept* A is normatively *incoherent* (“out of bounds”)—a constellation of commitments to which one *cannot* be entitled (entitlement is precluded).

⁵ Ulf Hlobil “The Laws of Thought and the Laws of Truth as Two Sides of One Coin” [ROLE: July 1, 2021]. [Update [ref.] as needed.]

I believe that this isomorphism between the definitions of reason relations of implication and incompatibility in the bilateral semantic framework of verifiers and falsifiers and the bilateral pragmatic framework of acceptance and rejection is deep and revealing. To begin with, it shows how the connection between two paired truth values and two paired doxastic attitudes expressed in the principles that accepting is taking-true and rejecting is taking-false is reflected, and can be further elaborated at the level of the reason relations of implication and incompatibility that articulate the contents that can *be* true/taken-true and false/taken-false. In particular, substantial new light is shed on what one must *do* to count thereby *as* adopting a practical attitude of taking some claimable to be true or false when those attitudes are situated in the wider context of practices of giving reasons for and against claimables that are constrained by reason relations of implication and incompatibility. The isomorphic relation between what is expressed by semantic metavocabularies of truth-makers and false-makers and what is expressed by pragmatic metavocabularies of bilateral commitments and preclusions of entitlement clarifies the relations between what one is *saying* and what one must be *doing* in order to say that in using the object language those semantic and pragmatic metavocabularies address. In practically acknowledging that commitment to accept some claimables precludes entitlement to reject some others and to accept still others, practitioners are, we can now see, *thereby* taking it that the fusion of verifiers of the premises and falsifiers (respectively, verifiers) of the conclusions are impossible states.

Alethic modal relations of possibility, impossibility, and necessity are part of the essential structure of the worldly states and situations that, according to the truth-maker semantic model, *make* claimables true or false, and so are what is represented and talked *of* or thought *about* by the use of declarative sentences. *Deontic normative* relations of commitment, entitlement, and preclusion of entitlement are part of the essential structure of discursive practical attitudes adoption of which, according to the pragmatics-first model, is what practitioners must *do* in order thereby to count as taking or treating what is expressed by declarative sentences *as* true or false, thereby representing the world as being some ways and not others by saying or thinking *that* things are thus-and-so. The very same reason relations of implication and incompatibility, which articulate the claimable contents expressed by declarative sentences, what can both *be* true or false and be practically *taken* to be true or false by affirming or denying them, can be construed

equally and isomorphically both semantically, in alethic modal terms of *making* true or false, and *pragmatically*, in deontic normative terms of the practical doxastic attitudes of *taking* true or false (accepting or rejecting).

In *A Spirit of Trust* I attribute a view recognizably of this shape to Hegel, under the rubric “bimodal hylomorphic conceptual realism.” He emphasizes reason relations of material incompatibility (Aristotelian contrariety) over those of implication or material consequence—his notion of “determinate negation” over his notion of “mediation”—though both are always in play. As I read him, Hegel begins with the thought that ways the world can objectively be, facts, are determinate just insofar as they exclude and entail one another in a way properly expressed in alethic modal terms. That the coin is copper makes it *impossible* that it remain solid at 1100 degrees Celsius and *necessitates* its being an electrical conductor. By contrast, our subjective takings of the world to be some way, thoughts, are determinate just insofar as they exclude and entail one another in a way properly expressed in deontic normative terms. As I’ve suggested here that we put this point, my commitment to the coin’s being copper precludes *entitlement to accepting* that it would remain solid at 1100 degrees Celsius and precludes *entitlement to rejecting* that it is an electrical conductor. One and the same determinate conceptual content, that the coin is copper, can take two forms, an objective one in which it is understood as articulated by relations of exclusion and consequence construed in the alethic modal vocabulary proper to the expression of laws of nature, and a subjective one in which it is understood as articulated by relations of exclusion and consequence construed in the deontic normative vocabulary proper to the expression of discursive practices. That is why I use the term “bimodal hylomorphism.” The view is properly denominated conceptual “realism” because the very same conceptual content to which we adopt attitudes in thought is understood as present, albeit in a different form, in the objective world thought about. The world is accordingly construed as essentially always already in a thinkable shape.

The isomorphism Hlobil has worked out between Restall and Ripley’s normative pragmatic bilateral construal of implication and incompatibility relations and a version of Fine’s truth-maker semantics is a colorable contemporary development of a thought cognate to the bimodal hylomorphic conceptual realism I attribute to Hegel. It suggests how something like

this thought can be worked out in detail. For it maps onto one another a semantic idiom of great expressive power and flexibility and a pragmatic idiom that has shown its substantial utility in understanding sequent calculi. Each has been used to characterize the fine structure of reason relations in actual applications to multifarious different object vocabularies.

III. Implicational Phase-Space Semantics and the Truth-Maker Framework

When I introduced the idea of a pragmatics-first order of explanation, which would start with practices of accepting and rejecting and giving and asking for reasons entitling one to adopt those attitudes (so, challenging and defending doxastic commitments), I held out the prospect of a recognizably semantic understanding of the claimables that can be accepted or rejected (taken to be true or false) made available in terms of the reason relations of implication and incompatibility those claimables stand in to one another. We have seen how such reason relations can be understood in normative pragmatic terms of commitment and (preclusion of) entitlement, and how those very same reason relations can be reconstructed in paradigmatically semantic terms of worldly states or situations taken to make claimables true or false. But although the truth-maker semantics underwrites both a notion of the content expressed by declarative sentences and reason relations of implication and incompatibility that can also be understood in a normative pragmatic metavocabulary, it does not explain truth-evaluable content by appealing to those reason relations. Rather, it explains both in terms of modalized spaces of worldly states verifying and falsifying claimables. Striking as the isomorphism is that Hlobil points out and works out between truth-maker semantic construals of implication and incompatibility and normative pragmatic construals of them, it does not amount to an explanation of claimable content *by means of* reason relations. So it does not by itself count as redeeming the promissory note I issued on behalf of a pragmatics-first order of semantic explanation.

To do that we can look to the implicational phase-space semantics (IPSS) developed by Dan Kaplan, a Pittsburgh member of our ROLE logic working group. It implements precisely what I have been promising: an understanding of what is expressed by declarative sentences in terms of the role those sentences play in reason relations of implication and incompatibility. In so doing it fulfills the defining aspiration of the philosophical tradition I call “semantic inferentialism.” It begins with what I regard as a remarkable conceptual innovation. Not only

are the semantic interpretants it appeals to implications (and incompatibilities), so is what is interpreted. That is, the principal and original bearers of semantic significance are construed not as sentences, but as implications.

The points of an implicational phase space are *candidate implications* defined on a language L_0 thought of as a set of logically atomic sentences. The candidate implications are then all ordered pairs $\langle \Gamma, \Delta \rangle \in L_0 \times L_0$ of sets of sentences of the language. They are what we have been representing by statements formed using the snake turnstile “ $\Gamma | \sim \Delta$.” This is the sort of thing manipulated in proof-theoretic multisuccedent sequent calculi—and given normative pragmatic interpretations by Restall-Ripley bilateralism.

As is usual in such calculi, incompatibility is represented by empty right-hand sides rather than by a distinctive sort of turnstile: “ $\Gamma, A | \sim$ ” rather than “ $\Gamma \# A$ ”. (The empty right-hand side marks the incoherence of the set of premises that appears on the left-hand side of the turnstile.)

I call the points of the implicational phase space “candidate” implications because they do not represent *good* implications: just candidates for that status. The good implications, the ones that actually hold—intuitively, where the set on the right-hand side, taken disjunctively, is a genuine consequence of the set of premises on the left-hand side, taken conjunctively, are marked as members of a distinguished subspace \mathbf{I}_0 of *good* implications.

The third element of an implicational phase-space semantic model for a language L_0 —in addition to the space of candidate-implication points $L_0 \times L_0$ and the subspace of good implications \mathbf{I}_0 —is an operation \cup of *adjunction* of candidate implications.

It is defined by:

Adjunction: $\langle \Gamma, \Delta \rangle \cup \langle \Phi, \Lambda \rangle =_{\text{df.}} \langle \Gamma \cup \Phi, \Delta \cup \Lambda \rangle$.

To adjoin two candidate implications one produces a third candidate implication by combining (in the sense of unioning) their premises and combining (in the sense of unioning) their conclusions. With the minimal candidate implication $\langle \emptyset, \emptyset \rangle$ playing the role of an identity element, adjunction is a commutative monoid on the space $L_0 \times L_0$.

Each candidate implication can now be assigned, as its semantic interpretant, the set of candidate implications whose adjunctions with it yield good implications, implications in the distinguished set \mathbf{I}_0 .

Υ -sets: $\forall x \in L_0 \times L_0 \quad x^\Upsilon =_{\text{df.}} \{y \in L_0 \times L_0 : x \cup y \in \mathbf{I}_0\}$.⁶

The Υ -set (pronounced “vee set”) of a candidate implication $\langle \Gamma, \Delta \rangle$ is what you need to add (adjoin) to it to get a *good* implication. If $\langle \Gamma, \Delta \rangle$ is already a good implication (if it is in \mathbf{I}_0) that fact will be marked by the fact that the minimal candidate implication $\langle \emptyset, \emptyset \rangle$ will be in its Υ -set. If $\langle \Gamma, \Delta \rangle$ is a good implication, its Υ -set $\langle \Gamma, \Delta \rangle^\Upsilon$ is something like its *range of subjunctive robustness*. Focusing for simplicity on the premise-set Γ , the Υ -set is telling us what further collateral premises we can add to it without infirming the implication: turning it from a good one to a bad one. If the hungry lioness sees a limping gazelle nearby, then she will pursue it. That implication would still be good even if the beetle on a distant tree climbs a bit further out on the branch as it is sitting on. But it would not be good if the lioness were suddenly struck by lightning. If the candidate implication is not a good one, its Υ -set tells us what we would need to add (adjoin) to it to *make* it a good one. Intuitively, the Υ -sets play a role with respect to implications that is analogous to the role played by truth conditions with respect to sentences. They both specify what it would take for one to be semantically good—in the (different) ways implications and sentences can be semantically good.

At a second, separate stage, this semantic interpretation *of* (sets of) implications *by* sets of implications can then be extended to specify the semantic roles played by *sentences in* implications (and incompatibilities), rather than just of the implications themselves. In this implications-first inferentialist setting, a sentence A can be represented for semantic purposes by a pair of implications: $\langle \langle A, \emptyset \rangle, \langle \emptyset, A \rangle \rangle$. The semantic content expressed by the sentence—in the sense of its role in reason relations of implication and incompatibility—can then be represented by the Υ -sets of these paired implications. $\langle A, \emptyset \rangle^\Upsilon$ determines the set of all the good implications in which A figures as a premise. $\langle \emptyset, A \rangle^\Upsilon$ determines the set of all the good

⁶ Υ -sets can be computed for *sets* of implications by requiring that each element of the Υ -set yield an element of \mathbf{I} when adjoined with *every* element of the set: $\forall X \subseteq L_0 \times L_0 \quad X^\Upsilon =_{\text{df.}} \{y \in L_0 \times L_0 : \forall x \in X [x \cup y \in \mathbf{I}_0]\}$.

implications in which A figures as a conclusion. For each tells us what additions to the bare skeletons of $\langle A, \emptyset \rangle$ and $\langle \emptyset, A \rangle$ yield good implications. The nature of the adjunction operation, in terms of which (together with the set of good implications \mathbf{I}_0) the γ -sets are defined, ensures that A appears as a premise in every element of the set of good implications that results from adjoining elements of $\langle A, \emptyset \rangle^\gamma$ to $\langle A, \emptyset \rangle$, and as a conclusion in every element of the set of good implications that results from adjoining elements of $\langle \emptyset, A \rangle^\gamma$ to $\langle \emptyset, A \rangle$.

The claim is that broadly inferential roles, in the sense specified by pairs of premissory and conclusory γ -sets $\langle \langle A, \emptyset \rangle^\gamma \langle \emptyset, A \rangle^\gamma \rangle$ are a good representation of what one must grasp in order to understand what one is accepting or rejecting in undertaking doxastic commitments.⁷ For it is inferential roles in this sense that determine what is a reason for and against the claims to which one is committing oneself, and so what it would take to entitle oneself to those attitudes and the acts of affirmation and denial that overtly manifest them. For that reason, these are good semantic representations of the claimable contents expressed by declarative sentences. Of course, the idea is not that in order to defend and challenge doxastic commitments we need to have fully mastered the intricacies of these inferential roles. It is that insofar as we do not, we do not know what we are committing ourselves to, do not fully understand what we are accepting or rejecting, or the reasons we give entitling us to do so.

A minimal criterion of adequacy for Kaplan's implicational phase-space semantics is that it can be shown to offer a tractable semantics for the logically complex sentences that result when we extend the logically atomic language L_0 by introducing sentential logical vocabulary according to a wide variety of sequent rules. Indeed, Kaplan proves soundness and completeness results using the implicational phase-space semantics for a number of such logics, including not only classical and intuitionistic logics, but also a wide variety of substructural (nonmonotonic, nontransitive, noncontractive...) logics). This broadly inferentialist semantic account of the claimable (acceptable/rejectable) contents expressed by declarative sentences is what I had in mind when I initially raised the possibility that a pragmatics-first approach that understands reason relations of implication and incompatibility in normative terms of what one is *doing* in

⁷ For both conceptual and technical reasons, it turns out that it is best to use the closures of these γ -sets under the γ -function, which can be shown to reach a fixed point at $\langle \langle A, \emptyset \rangle^{\gamma\gamma} \langle \emptyset, A \rangle^{\gamma\gamma} \rangle$, but I suppress this complication.

adopting doxastic practical attitudes of accepting and rejecting claims and challenging and defending entitlement to the resulting commitments by offering reasons for and against them could be built on, extended, and developed to provide an adequate semantics.

I have gestured at two routes to semantics. I have described how Hlobil offers a way of understanding (his version of) reason relations, paradigmatically implication, in Fine’s truth-maker semantics, in terms of an isomorphism with Restall and Ripley’s bilateralist normative pragmatics. I have explained in general terms how Kaplan defines his implicational phase-space semantics directly in terms of implication (and incompatibility) relations, which we have seen can be understood in normative pragmatic terms of acceptance and rejection, commitments and (preclusions of) entitlement. I want to close by comparing and contrasting the reconstructions of reason relations of implication and incompatibility offered by these two semantic approaches: in terms of truth-makers and in terms of implications.

The first thing to appreciate is the strong formal analogies between the two frameworks. The modalized state spaces of truth-maker semantics are built on sets of “states” that are not further specified. The states making up these spaces could be pretty much anything—which of course contributes greatly to the expressive flexibility of the apparatus. Within the set S of states, a privileged subset of “good” ones, S^\diamond is distinguished—intuitively, by its alethic modal status as “possible.” Kaplan’s implicational phase spaces are sets of points that have more structure than Fine’s states. They are candidate implications, pairs of sets of sentences drawn from an antecedent prelogical language. Within this space $L_0 \times L_0$ of implications, a privileged subset of “good” ones, I_0 is distinguished—intuitively, by its normative status as codifying the proper implications, what really follows from what. The operations on states or candidate implications, fusion \sqcup and adjunction \cup (the one stipulated, the other defined in terms of the additional structure of the space of candidate implications defined on L_0) are algebraically both commutative monoids.⁸ The semantic interpretants of sentences are in both cases bilateral: verifiers/falsifiers and premissory and conclusory \vee -sets respectively.

⁸ Both Fine’s truth-maker semantics on modalized state spaces and Kaplan’s implicational phase-space semantics use commutative monoids (the fusion/adjunction operation, together with a null space unit element) on spaces with distinguished subspaces (S^\diamond and I). This is an algebraic generalization of more familiar residuated lattices. In making this generalization, both are downstream from Girard’s phase-space semantics for linear logic.

There is also a substantial formal *difference* between the two settings. In the truth-maker framework, the modalized state space with its fusion operation (or part-whole relation among states) is wholly distinct from the language it is used to interpret semantically. To get a semantic model, a third element is required: an interpretation function that maps sentences of the language onto pairs of sets of verifying and falsifying states. In the implicational phase-space framework, there is nothing corresponding to this extra element, connecting and mediating between the language and the space on which it is interpreted. The extra structure that the points of the implicational phase-space come with, their being candidate implications in the form of pairs of sets of sentences of the language, not only means that the monoidal operation of adjunction of candidate implications can be explicitly defined set-theoretically, as opposed to simply stipulated, as with fusion of states. Because the sentences themselves are already present in the space from which semantic interpretants are drawn, the \vee -function that semantically interprets first implications and then sentences can also be explicitly defined set-theoretically from the raw materials already present in the implicational phase-space itself. In this sense, the interpretation function connecting sentences to their semantic interpretants is *intrinsic* to the sentences as they figure in the space of implications. The sentences come already interpreted by the reason relations they stand in to one another, the roles they play in implications and incompatibilities. All the semantic framework does is draw that implicit intrinsic interpretation out explicitly. Now whether this is a virtue or a vice, a benefit or a cost, will depend on collateral theoretical commitments. For one might see it as showing that the implicational phase-space framework is foolishly trying to do without relations to extralinguistic reality that are what make truth-maker semantics a genuine *semantics* in the first place. I am not going to argue about that. But I do want to assemble some further considerations that might bear on such a dispute.

For in spite of the substantial difference in the conceptions of semantic interpretation that animate the two different approaches, the fact that both take the mathematical form of commutative monoids plus distinguished subspaces means that their treatment of the crucial reason relations of implication and incompatibility share enough structure to be intertranslatable across the two settings. That is, we can specify exactly the same reason relations of implication

and incompatibility while moving systematically between the modalized state spaces of truth-maker semantics and implicational phase-space semantics. Here's how.

For one direction: Beginning with a truth-maker model, one can define an implicational phase space that corresponds to it in the sense of defining exactly the same implications and incompatibilities. We are given a truth-maker model of a language L_0 , defined on a modalized state space $\langle S, S^\diamond, \sqcup \rangle$, which assigns to each sentence $A \in L_0$ a pair of sets of states $\langle v(A), f(A) \rangle$ understood as verifiers and falsifiers of that sentence. The points of the implicational phase space being defined are ordered pairs of sets of sentences of L_0 . These are the candidate implications. What corresponds to fusion, \sqcup , is adjunction:

$$\langle \Gamma, \Delta \rangle \sqcup \langle \Theta, \Psi \rangle = \langle \Gamma \cup \Theta, \Delta \cup \Psi \rangle,$$

as usually defined in implicational phase space semantics. It remains to compute \mathbf{I}_0 , the set of *good* implications. We do that using the consequence relation H_{lobil} defined to mimic the Restall-Ripley bilateral understanding of the multisuccedent turnstile:

$$\langle \Gamma, \Delta \rangle \in \mathbf{I}_0 \text{ iff } \forall s, t \in S [(\forall G \in \Gamma [s \in v(G)] \ \& \ \forall D \in \Delta [t \in f(D)]) \Rightarrow s \sqcup t \notin S^\diamond].$$

That is,

$\langle \Gamma, \Delta \rangle$ is a good implication just in case the fusion of any state s that verifies all of Γ and any state t that falsifies all of Δ is an impossible state, in the truth-maker model. This construction obviously guarantees that exactly the same implications will hold in the implicational phase space, that is, be elements of \mathbf{I}_0 , as satisfy the H_{lobil} consequence relation in the truth-maker model.

As for incompatibilities, in the truth-maker setting, two *states* s and t are incompatible just in case their fusion is an impossible state. Two *sentences* A and B are incompatible just in case any fusion of a verifier of the one with a verifier of the other is an impossible state. More generally, a set Γ of sentences is *incoherent* in case any fusion of verifiers of all its elements is an impossible state. Given the definition of the set of good implications \mathbf{I}_0 just offered, this is equivalent to $\langle \Gamma, \emptyset \rangle \in \mathbf{I}_0$. The incompatibilities are represented in the implicational phase space semantics just by good implications with empty right-hand sides.

So there is a straightforward method for taking any truth-maker model defined on a modalized state space and defining from it an implicational phase space model that has exactly the same reason relations of implication and incompatibility.

It turns out that one can also go in the other direction: Beginning with an implicational phase space, one can define a truth-maker model (an interpreted modalized state space) that corresponds to it in the sense of defining exactly the same implications and incompatibilities.

I won't go through the details here.

We are given an implicational phase space defined on a language L_0 , $\langle \mathcal{P}(L_0) \times \mathcal{P}(L_0), \mathbf{I}_0 \rangle$. The states will be candidate implications. $S = \mathcal{P}(L_0) \times \mathcal{P}(L_0)$. \sqcup is adjunction: $\langle \Gamma, \Delta \rangle \sqcup \langle \Theta, \Psi \rangle = \langle \Gamma \cup \Theta, \Delta \cup \Psi \rangle$. In the Hlobil truth-maker definition of consequence, the *good* implications correspond to *impossible* states. So the subset of *possible* states is defined by $S^\diamond = S - \mathbf{I}_0$.

It remains to define the model function m , which assigns to each $A \in L_0$ a pair of subsets of S , $\langle v(A), f(A) \rangle$, where $v(A) \subseteq L_0$ and $f(A) \subseteq L_0$, such that:

$$\langle \Gamma, \Delta \rangle \in \mathbf{I}_0 \text{ iff } \forall s, t \in S [(\Gamma = \{G_1 \dots G_n\} \ \& \ g_1 \in v(G_1) \ \& \dots \ g_n \in v(G_n) \ \& \ s = g_1 \sqcup \dots \sqcup g_n \ \& \ \Delta = \{D_1 \dots D_n\} \ \& \ d_1 \in v(D_1) \ \& \dots \ d_n \in v(D_n) \ \& \ t = d_1 \sqcup \dots \sqcup d_n) \Rightarrow s \sqcup t \notin S^\diamond].$$

For various metatheoretic purposes, Fine employs “canonical” truth-making models, in which the verifier of a (logically atomic) sentence is just that sentence and the falsifier of that sentence is just the negation of that sentence. (His requirement that the fusion of any verifiers of A will be a verifier of A and the fusion of any falsifiers of A will also be a falsifier of A is then trivially satisfied, since there is only one.) We can combine that idea with Kaplan’s standard representation of the proposition expressed by A as the pair $\langle \langle A, \emptyset \rangle, \langle \emptyset, A \rangle \rangle$, and do without the formation of falsifying literals by appeal to negation by defining the verifiers of A by $v(A) = \langle A, \emptyset \rangle$ and the falsifiers of A by $f(A) = \langle \emptyset, A \rangle$.

We want to implement Hlobil’s definition of implication (which generalizes C. I. Lewis’s strict implication to Fine’s truthmaker semantic framework), that an implication $\Gamma \sim \Delta$ is good in the truth-maker setting just in case the fusion of any verifier of all of Γ and any falsifier of all of Δ is an impossible state. To do that, we need to say what it is for a state (defined in the implicational phase space, that is, a candidate implication) to “verify all of Γ ” and to “falsify all of Δ .” We can extend the single-sentence definitions as follows. If $\Gamma = \{G_1 \dots G_n\}$ and $\Delta = \{D_1 \dots D_m\}$:

$$v(\Gamma) = \langle \Gamma, \emptyset \rangle = \langle G_1, \emptyset \rangle \cup \dots \cup \langle G_n, \emptyset \rangle.$$

$$f(\Delta) = \langle \emptyset, \Delta \rangle = \langle \emptyset, D_1 \rangle \cup \dots \cup \langle \emptyset, D_m \rangle.$$

That is, the implication (standing in for a state) $\langle \Gamma, \emptyset \rangle$ counts as verifying all of Γ because it is the adjunction of the verifiers of each element of Γ . (In this “canonical” modalized state-space model, sets of sentences, like individual sentences, only have single states=implications as verifiers.) And similarly for falsifiers.

To show that this works, in the sense of yielding the same implications in the truth-maker model that are good in the original implicational phase space, we must show that

$$\langle \Gamma, \Delta \rangle \in \mathbf{I}_0 \text{ iff } \forall s, t \in S [(\forall G \in \Gamma [s \in v(G)] \ \& \ \forall D \in \Delta [t \in f(D)]) \Rightarrow s \sqcup t \notin S^\diamond].$$

To show the left-to-right direction \Rightarrow : If $\langle \Gamma, \Delta \rangle \in \mathbf{I}_0$ then $v(\Gamma) = \langle \Gamma, \emptyset \rangle$ and $f(\Delta) = \langle \emptyset, \Delta \rangle$. So $v(\Gamma) \sqcup f(\Delta) = \langle \Gamma, \Delta \rangle$.

Since by hypothesis $\langle \Gamma, \Delta \rangle \in \mathbf{I}_0$, by the definition of S^\diamond as $S - \mathbf{I}_0$, it follows that $\langle \Gamma, \Delta \rangle \notin S^\diamond$, that is, that the state $\langle \Gamma, \Delta \rangle$ is an impossible state. It is the fusion of *the* verifier of Γ , $\langle \Gamma, \emptyset \rangle$ and *the* falsifier of Δ $\langle \emptyset, \Delta \rangle$ because it is the result of adjoining them.

To show the right-to-left direction \Leftarrow : If $\forall s, t \in S [(\Gamma = \{G_1 \dots G_n\} \ \& \ g_1 \in v(G_1) \ \& \dots \ g_n \in v(G_n) \ \& \ s = g_1 \sqcup \dots \sqcup g_n \ \& \ \Delta = \{D_1 \dots D_n\} \ \& \ d_1 \in v(D_1) \ \& \dots \ d_n \in v(D_n) \ \& \ t = d_1 \sqcup \dots \sqcup d_n) \Rightarrow s \sqcup t \notin S^\diamond]$, then $s = v(\Gamma)$ and $t = f(\Delta)$, so $v(\Gamma) \sqcup f(\Delta) = \langle \Gamma, \Delta \rangle \notin S^\diamond$. Since $S^\diamond = S - \mathbf{I}_0$ and $\langle \Gamma, \Delta \rangle \in S$, $\langle \Gamma, \Delta \rangle \in \mathbf{I}_0$.

As for incompatibility, we must show that A and B are truth-maker incompatible (Γ is truth-maker incoherent), that is, $\forall s, t \in S [s \in v(A) \ \& \ t \in v(B) \Rightarrow s \sqcup t \notin S^\diamond]$, (or more generally, $v(\Gamma) \notin S^\diamond$ iff $\langle \{A, B\}, \emptyset \rangle \in \mathbf{I}_0$ (or more generally, $\langle \Gamma, \emptyset \rangle \in \mathbf{I}_0$).

To show the left-to-right direction \Rightarrow : If $\forall s, t \in S [s \in v(A) \ \& \ t \in v(B) \Rightarrow s \sqcup t \notin S^\diamond]$, then since $v(A) = \langle A, \emptyset \rangle$ and $v(B) = \langle \emptyset, B \rangle$, and since \sqcup is adjunction, $s \sqcup t = \langle \{A\} \cup \{B\}, \emptyset \rangle = \langle \{A, B\}, \emptyset \rangle$. Since $\Rightarrow s \sqcup t \notin S^\diamond$, $s \sqcup t = \langle \{A, B\}, \emptyset \rangle \in \mathbf{I}_0$.

This works for arbitrary iterations of \sqcup , which gives the more general Γ case.

To show the right-to-left direction \Leftarrow : If $\langle \{A, B\}, \emptyset \rangle \in \mathbf{I}_0$, then $\langle \{A\} \cup \{B\}, \emptyset \rangle \in \mathbf{I}_0$.

Since \sqcup is adjunction, $\langle A, \emptyset \rangle \sqcup \langle \emptyset, B \rangle \in \mathbf{I}_0$. But $v(A) = \langle A, \emptyset \rangle$ and $v(B) = \langle \emptyset, B \rangle$.

So $v(A) \sqcup v(B) \in \mathbf{I}_0$. Since $S^\diamond = S - \mathbf{I}_0$, $v(A) \sqcup v(B) \notin S^\diamond$. That is truth-maker incompatibility of A and B. This works for arbitrary iterations of \sqcup , which gives the more general Γ case.

The result is a straightforward uniform translation between Kaplan's implicational phase-space semantics and Fine's truth-maker semantics. Each truth-maker model on a language corresponds to an implicational phase-space defined on that same language, in the sense that they underwrite exactly the same reason relations of implication and incompatibility.

The parallel extends to various structural constraints that can be placed on them—Fine's Exclusivity, Downward Closure, and Exhaustivity conditions, which I'll have more to say about further along.

This translation shows how truthmaker semantics can be “deflated” from the point of view of semantic inferentialism. For it shows how to extract what the inferentialist insists is its semantic core: the way it functions to codify reason the relations of implication and incompatibility that articulate claimable (so, propositional) contents. The representational, metaphysical reading of “truthmaking states” is, from this perspective, optional and inessential:

at best a harmless indulgence, at worst a misleading characterization of the semantic enterprise. The position that results is the extension to the more sophisticated truthmaking and implicational phase-space semantics of the inferentialists views about classical model-theory and proof-theory. Both are seen as providing metavocabularies for codifying reason relations of implication and incompatibility. In the classical case, the differences in the expressive power of representational and inferential metavocabularies is interesting and instructive, but not a reason to see one or the other approach as simply wrong-headed. The isomorphism between truthmaking and implicational phase-space semantics (the latter accompanied by, and sound and complete with respect to, a powerful sequent calculus) should engender the same irenic attitude toward these semantic metavocabularies.⁹

⁹ I have been talking about how the “internal” consequence (and incompatibility) relations line up in the two settings. Looking somewhat further afield, the deep affinities between these two semantic approaches are also manifested in the way verifiers line up with premissory roles and falsifiers with conclusory roles, in the *external* consequence relations. (The internal relations cross the turnstile(s). The external ones remain on one side of the turnstile, looking at relations between the premissory sides of different sequents, or between the conclusory sides of different sequents. In substructural cases, the internal and external consequence relations can diverge.) Kaplan shows that K3 (the Strong Kleene three-valued logic) is the unilateral external logic of premissory roles in codifying the sense of consequence in which $A \models_p B$ just in case if in the internal logic $\Gamma, B \vdash \Delta$ then $\Gamma, A \vdash \Delta$ (A can replace B as a premise, saving the goodness of implications), and LP (Graham Priest’s “Logic of Paradox”) is the unilateral external logic of conclusory roles in codifying the sense of consequence in which $A \models_c B$ just in case if in the internal logic $\Gamma \vdash A, \Delta$ then $\Gamma \vdash B, \Delta$ (B can replace A as a conclusion, saving the goodness of implications). Hlobil shows that K3 is the unilateral external “logic of verifiers,” in the sense that K3 preserves compatibility with the verifiers of the premises (jointly) to the verifiers of the conclusions (separately). And Hlobil shows that LP is the unilateral “logic of falsifiers,” in the sense that LP preserves the compatibility potential of the falsifiers of the conclusions (jointly) to the falsifiers of the premises (separately). So the isomorphism between the reason relations specified by the truth-maker semantics and those specified by the implicational phase-space semantics goes beyond the internal (bilateral) consequence relations all the way to the external (unilateral) consequence relations as well.

Conclusion

I have sketched an order of explanation that moves from pragmatics to semantics. The most basic use of linguistic expressions is to perform speech acts of assertion and denial, manifesting doxastic attitudes of accepting and rejecting. I claimed that what makes the practical attitudes *doxastic* attitudes (and so makes the locutions that express them *declarative sentences*) is their standing liability to *challenges* by assertions that provide reasons *against* them, and the consequent obligation to *defend* them by assertions that provide reasons *for* them. Those dialogic practices make intelligible reason relations of implication and incompatibility, which can be understood in terms of normative statuses of *commitment* to accept and reject and (preclusion of) *entitlement* to such commitments. The second stage of the envisaged pragmatics-first order of explanation then semantically characterizes the claimable contents expressed by the declarative sentences that are asserted or denied, what can be doxastically accepted or rejected, in terms of the functional roles those sentences play in reason relations of implication and incompatibility. Dan Kaplan's substructural implicational phase-space semantics shows in detail how an expressively powerful formal semantics can be elaborated from the material relations of implication and incompatibility that precipitate out of the functionalist story told in such a normative pragmatic metavocabulary. This is the principal story I want to put on the table.

An exciting recent result of Ulf Hlobil's shows that Kit Fine's truthmaker semantics stands in surprising relations to the normative pragmatics gestured at here, and I build on that result to show that it also stands in surprising relations to Kaplan's inferentialist semantics. As a secondary project, I have sketched how those results can be used to facilitate the comparison of the pragmatics-first order of explanation with the most sophisticated contemporary development of the semantics-first order of explanation.

For we see first that the pragmatic significance of relations of implication and incompatibility defined in truthmaker terms can be articulated in bilateralist terms of the Restall-Ripley sort, and, by extension, in terms of normative statuses of commitment and entitlement suggested here to sharpen their account. This new way of building a pragmatics on top of

truthmaker semantics marks a fault line, or at least a division of labor, within the truthmaker setting. On one side, there is the metaphysical story about states, about their mereological fusion, and about the division of them into possible and impossible. On the other side there are the reason relations of implication and incompatibility that Hlobil shows how to define on that basis. The pragmatic connection to discursive practices of defending and challenging doxastic attitudes of acceptance and rejection depends *only* on the latter. This is the only part of the semantic story that shows up as pragmatically significant. To show that the metaphysics matters to (or, further, is even implicit in) the *use* of the expressions to whose meaning it purports to contribute, some further pragmatic story would have to be told, going beyond the one envisaged here.

Further, we saw that the meaning relations of implication and incompatibility generated by the heavily metaphysically committive truthmaker semantics can be reproduced exactly within the much less metaphysically committive implicational phase-space semantics, all of whose primitives can be understood as implicit in the pragmatics that specifies the use of expressions in discursive practice. So, space is opened for a *deflated* version of the truthmaker semantics. The claim here would be that the commutative monoid that does the heavy lifting in truthmaker semantics has been mischaracterized, needlessly and misleadingly encrusted with functionally irrelevant baroque metaphysical ornamentation. Underneath that misleading guise, what is doing the work is Kaplan's operator adjoining premises and conclusions of implications to mark their ranges of subjunctive robustness.¹⁰

Let me close with the observation that the very same normative pragmatic metavocabulary—of commitments to accept or to reject, and of preclusion of entitlement to such commitments—that can be used to specify the reasoning practices in which sentences of the prelogical object language are used to make, challenge, and defend claims, *also* suffices to specify the use of truth-first semantic metavocabularies (including the Fine's hyperintensional truthmaker version) to characterize both reason relations of implication and incompatibility and what is expressed by the declarative sentences that can be accepted or rejected, true or false. For the pragmatic

¹⁰ It should be acknowledged that the isomorphism with Kaplan's implicational phase-space semantics has been shown to hold only when consequence in the truthmaker setting is defined the way Hlobil does in order to map that semantic setting onto Restall-Ripley bilateralist normative pragmatics. This is not how Fine himself defines consequence. He considers and employs a variety of such definitions, but taking an implication to be good if and only if the fusion of truthmakers of all the elements of the premise set and falsemakers of all the elements of the conclusion set is an impossible state, though natural enough, is not one of them.

metavocabulary for the truth-first semantic metavocabulary underlines the fact that what the semantic theorist is *doing* in sorting or evaluating claimables to begin with as true or false (perhaps guided by a view about what states would verify or falsify them) is just what the pragmatic metavocabulary takes as adopting the basic practical doxastic attitudes: *taking*-true (accepting) and *taking*-false (rejecting). The pragmatics-first order of explanation begins by explicitly theorizing about those practical attitudes as they show up in the use of the object language. The semantics-first order of explanation begins by practically adopting such attitudes, implicitly, and in an untheorized way, as part of the unexplained, taken-for-granted use of its semantic metavocabulary. The attitudes are fundamental in either case. The difference is just how theoretically and methodologically self-conscious one is about them. In the semantics-first order of explanation, the issue of what one is doing in making truth evaluations in the semantic metavocabulary, and in particular, what reasons entitle one to privilege *these* takings-true and takings-false (acceptances and rejections) is resolutely kept off-stage. This seems a point in favor of the pragmatics-first approach.

End